VALIDATION OF THE MAD9P ACCELERATION MODEL

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MAD9p is a completely object oriented, parallel C++ particle tracking program and has now been used for the first time to study accelerated beams in the Injector 2 cyclotron. First results concerning the center region are satisfactory and further beam dynamic analysis can soon be expected.

INTRODUCTION

MAD9P (**m**ethodical **a**ccelerator **d**esign version **9** - **p**arallel) is a general purpose parallel particle tracking program including 3D space charge calculation [1].

Mathematical and Physical Model

MAD9P is based on the Vlasov-Maxwell equations. In this model, particle motion is governed by external fields and a mean-field approach for the space-charge fields. Particle collisions and radiation are neglected. The total Hamiltonian for a beam line element can be written as a sum of two parts, $\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2$, which correspond to the external and space charge contributions. A second-order integration algorithm (split operator) for a single step is then given by

$$\mathcal{M}_k(\tau) = \mathcal{M}_k^1(\tau/2)\, \mathcal{M}_k^2(\tau)\, \mathcal{M}_k^1(\tau/2) + \mathcal{O}(\tau^3) \qquad (1)$$

where τ denotes the step size, $\mathcal{M}_k^1$ is the map corresponding to $\mathcal{H}_1$ obtained by differential algebra methods from a general relativistic Hamiltonian and $\mathcal{M}_k^2$ is the map corresponding to $\mathcal{H}_2$. $\mathcal{M}_k^2$ is obtained by discretizing the resulting Poisson problem on a rectangular mesh using Fourier techniques to solve the time consuming cyclic convolution in $\mathcal{O}(N \log N)$, where N is the number of grid points. Open and periodic boundary conditions can be chosen. In MAD9p are several beam line elements available: drift space, quadrupole, sectormagnetic bend, collimator, RF cavity and markers which save bunch informations like phase space and statistics. Once the physical elements are put together in an arbitrary way the elements are assumed to be perfectly aligned. To every beam element belongs a corresponding transfer map $\mathcal{M}_k$ which maps every initial condition ζ^i of the six dimensional phase space onto a final condition ζ^f by

$$\zeta^f = \mathcal{M}_k \zeta^i. \qquad (2)$$

MAD9p derives $\mathcal{M}$ by a *Lie algebraic method*. The fact that the negative Poisson Bracket of the Hamiltonian and the density function f is just the derivative of the density function with respect to the time leads to

$$f(t) = e^{-[t,\mathcal{H}]} \cdot f(0). \qquad (3)$$

$e^{[t,\mathcal{H}]}$ corresponds to $\mathcal{M}$ which can now be expanded in a Taylor series.

MODEL OF ACCELERATION

The acceleration element of MAD9p is a thin radio frequency (RF) cavity and can be specified as shown in the following example

```
C1:RFCAVITY, L=1.0, Volt=1.0e-3,
        LAG=PI/4, FREQ=50e6;
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from a MAD9p input file. This specifies a thin radio frequency cavity with length 1 meter, a peak potential drop of 1 mega volt, a phase shift (θ) of $\pi/4$ and a frequency of 50 MHz. The associated vector potential for the cavity reads:

$$A_z = -\frac{\hat{E}}{\omega} \cos(\omega t + \theta) \qquad (4)$$

with $\hat{E}$ the peak electric field. The thin radio frequency cavity is modeled in the impulse approximation, it is assumed that the cavity has an infinitely short active section, preceded or followed by a finite drift section. The active section has a longitudinal electric field component given by

$$E_z = \hat{E} \sin(\omega t + \theta). \qquad (5)$$

In the impulse approximation, the length L of the cavity is allowed to tend to zero in such a way that the product $E_0 L$ remains finite. A particle passing through the cavity gets an accelerating or decelerating change in energy, the magnitude beign dependent on the arrival time of the particle as shown in Fig. 1.

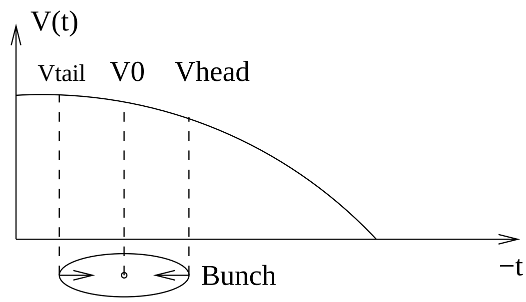

Fig. 1: Acceleration of a particle bunch with a sinusoidal field

VALIDATION OF THE RF CAVITY

All beam line elements of MAD9p but the RF cavity have been tested in [1]. Simple element configurations are considered in order to test the proper RF cavity implementation in MAD9p, specially the reference momenta (P_0) update. This is a crucial because in the map (3), $\mathcal{H}$ is scaled by P_0.

Generating the Lattice File for Injector 2

The next test is now done with an analytic model of Injector 2, aiming to model the collimation section as shown in Fig. 2. The Injector 2 cyclotron lattice file is constructed by a four fold symmetry hard-edge magnet description. After fixing the accelerator constants like RF and harmonic number the reference energy and the acceleration voltage are calculated as functions of the radius. Starting from these data the total circumference U of the static reference orbit can be calculated. The length of the orbit part inside the magnet, l_{hill}, is determined by the bending radius. The straight section between the magnets, l_{valley}, is the difference of a fourth of the circumference and l_{hill}. A Maple script is used to generate the entire lattice including measured RF cavity parameters and their interpolation. Additionally, the collimator's (special MAD9p elements) can be positioned. The flat top cavities are not included. The first $\pi/2$-arc - does not belong to the four fold symmetry and is treated separately.

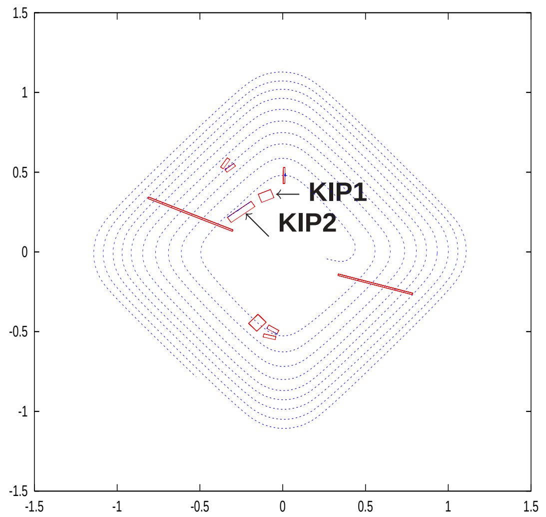

Fig. 2: Collimator layout of center region Injector 2

First results for the center region

To fix the non trivial initial conditions we start with one turn [1] and the estimated particle distribution from [1]. After lengthy precision work on positioning the collimators and finetuning details of injections we were able to simulate the very beginning of Injector 2 with satisfactory results. The amount of beam deposition on some collimators as well as the collimation process shown in Fig. 3 and Fig. 4 are well in agreement with observation and theory. The z-axis is the direction of beam propagation and the x-axis points to the center of the cyclotron. To analyse the immense amount of produced data (phase spaces of particles) several *root*-scripts [2] were written to compare the bunch shapes and intensities at different positions. Looking at Fig. 3 and Fig. 4

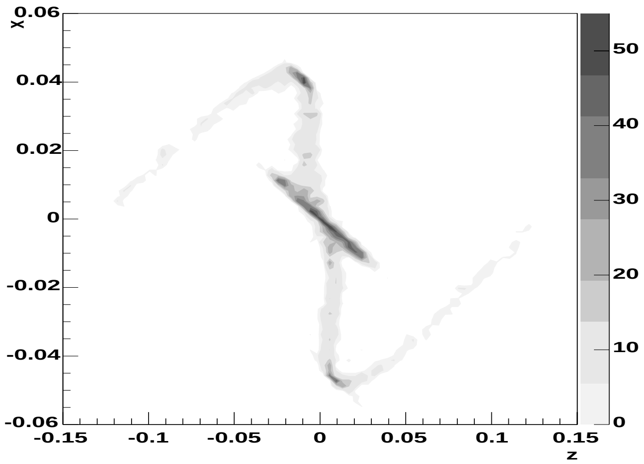

Fig. 3: Spatial particle density in a.u before KIP1

makes it clear that the bunch center rotates itself, the lower arm is expanding and the bunch has been collimated at the right place. The acceleration model is thus usable and promises detailed beam dynamic analysis in the near future.

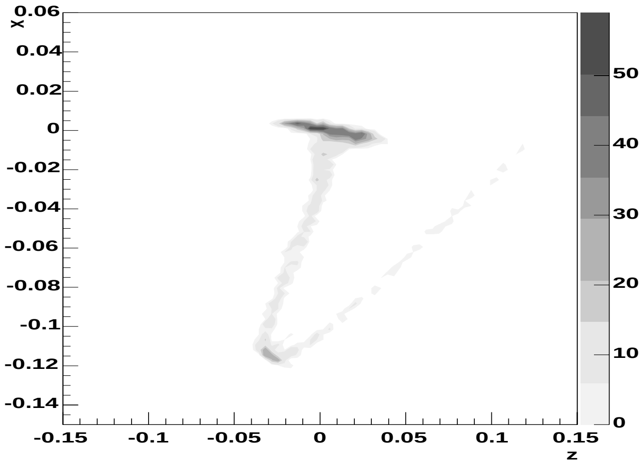

Fig. 4: Spatial particle density in a.u after KIP2

[1] A full Injector 2 simulation with 10^7 particles on 16 nodes (PSI Merlin Linux cluster) would last an entire weekend.

ACKNOWLEDGMENTS

Explanations about Injector 2 geometry from M. Humbel, S. Adam and the operator team are gratefully acknowledged.

REFERENCES

[1] Andreas Adelmann, *3D Simulations of Space Charge Effects in Particle Beams*, PhD Thesis ETHZ, October 2002, ISSN 1019-0643

[2] *www.root.cern.ch*, An object-oriented data analysis framework