REVIEW OF THE ROBINSON-STABILITY FOR THE RING-CYCLOTRON CAVITIES

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INTRODUCTION

The beam-loading model for storage rings was originally developed by K.W. Robinson [1]. For the case presented there, all the particle-bunches cross the cavity at the same location and the same energy, whereas in the case of the cyclotron they traverse the cavity at different radial positions and different energies. The knowledge of the cyclotron cavity-beam transfer function is the key for Robinson-stability analysis.

MODELLING THE INTERACTION

The action of the particles on the cavity modes can be calculated by Maxwell's equations using a mode-expansion method for the electric field and a Fourier decomposition of the flying particle bunches. The resulting differential equation of e.g. the fundamental mode is the same as for a lumped resonance circuit excited by the current source I_B, as shown in Fig. 1.

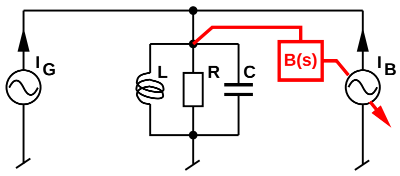

Fig. 1: RCL equivalence-circuit of cavity fundamental mode with beam-excitation I_B and generator I_G.

The contribution to the amplitude of the fundamental mode at the steady-state condition can be approximated by the sum over all turns at cyclotron radii r_t with velocity v_t and bunch-length σ_t

$$\Delta V_C = Z_C I_B = -Z_C \frac{2I_0}{\hat{G}} \sum_t G(r_t) T(v_t) L(\frac{\sigma_t}{v_t}) \quad (1)$$

for a cavity with gap-voltage distribution $G(r_t)$ and upper bound $\hat{G}$, impedance Z_C, transit-time correction $T(v_t)$ and long-bunch correction $L(\sigma_t/v_t)$ for an excitation with mean proton current I_0.

CAVITY-BEAM TRANSFER FUNCTION

The cavity-beam transfer function $B(s)$ describes the effect of a small phase or amplitude modulation of the cavity voltage V_C on the current I_B. A numerical method was used to calculate the variation in particle position and phase in response to the cavity voltage modulation.
The particle trajectory was integrated by a fourth order Runge-Kutta algorithm based on a third order Taylor-expansion of the static magnetic fields. The initial cavity voltages were adjusted to be in phase with $-I_B$ and to provide a particle end-energy of 590 MeV after 220 turns.

Zero beam-current trajectories were calculated for different initial phases of the modulation signal, cavity-crossings were determined and the variation in each interaction contribution (GTL_t in Eq. 1) was stored. The complex Fourier-coefficients were then evaluated for each interaction and superimposed after being phase shifted by the delay of the corresponding turn number. Fig. 2 shows the resulting transfer function of cavity voltage to current I_B in cavity 3. The first bump corresponds to the transit time of the beam in the cyclotron. The upward trend above 1 MHz is related to the revolution frequency at the sixth harmonic (8.4 MHz).

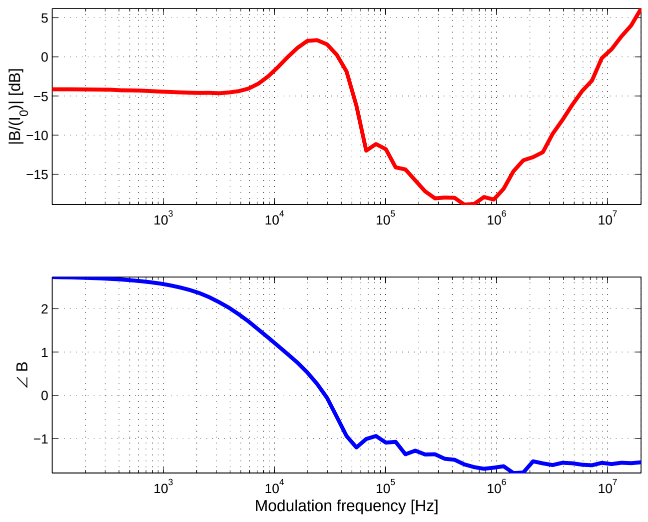

Fig. 2: Transfer function for phase-modulation.

CONCLUSIONS

Although closed-loop measurements of the system confirmed that no instability appears up to proton-beam currents of 1.9 mA averaged intensity, the cavity-beam transfer function could be the key for a future stability analysis of amplitude and phase control systems [3]. The method also allows calculation of the RF-power needs for the cyclotron. A comparison with the actually measured power indicated a deviation from the calculated value of about 10%. This will be the subject of further investigation.

REFERENCES

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