# SENSITIVITY ANALYSIS FOR COMET 

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#### Abstract

Asymmetry of the vertical coil position with respect to the median plane leads to beam losses in cyclotrons. This effect has been investigated for the PROSCAN cyclotron COMET. An analytical approach is compared with results from a simulation performed using the ray-tracing program TRACK. The effects on the beam position and on the beam size have been calculated.


## INTRODUCTION

In order to prepare for the commissioning of the cyclotron COMET, we have investigated the effects on particle trajectories caused by several possible distortions of the magnetic field. Here we report on effects due to a vertical shift of the coil.

## POSITION OF MEDIAN PLANE

In several cyclotrons, the vertical position of the main coil has been shown to have a strong impact on beam losses due to vertical oscillations [e.g. 2]. A vertical shift of the coil leads to a shift of the magnetic median plane (the plane where $B_{r}=0$ ), which could cause these particle losses.

The magnetic field can be split into the contribution from the iron and the one from the coil. We have extracted the two parts from the TOSCA model (scc_009_pre) obtained from the cyclotron manufacturer ACCEL Instruments GmbH . For this purpose, we have recalculated the model with $1 \%$ higher current. After subtracting the new field from the old (100 \% current) and multiplying the resulting field with 100 we "extracted" the coil field contribution. The difference between the total field and the coil field is then assumed to be the iron contribution. The two field contributions were saved in separate files and can be added with any desired relative spatial shift. This superposition is a simplified approach, but for small coil shifts it is a good approximation. A practical alternative does not really exist, as for some cases of coil asymmetry we would need to calculate full models and abandon the reduction to $1 / 8$, provided by the symmetry.
The azimuthally averaged fields derived with the method explained above are shown in Fig. 1a. From these, the radial field components of the two contributions have been calculated. In Fig. 1b, the radial field components at 1 mm distance from the median plane are plotted. As can be seen, up to $r=78 \mathrm{~cm} B_{r}$ is mainly coming from the coil. It can therefore be concluded that a vertical shift $\delta$ of the coil gives an approximately equal shift $Z$ of the median plane for $r<78 \mathrm{~cm}$.

An analytical expression of $Z(r)$ can be derived by inserting the median plane condition $B_{r}=0$ in the expression which describes the radial component of the field $B_{r}$ (averaged over the azimuth):

$$
\begin{equation*}
B_{r}(r, z) \approx z \cdot \frac{\partial B_{r}^{\text {iron }}}{\partial z}+(z-\delta) \cdot \frac{\partial B_{r}^{\text {coil }}}{\partial z} \tag{1}
\end{equation*}
$$

Using $\partial B_{r} / \partial z=\partial B_{z} / \partial r$, the vertical position of the magnetic median plane is then described as:

$$
\begin{equation*}
Z=Z_{B r=0}(r)=\delta \cdot \frac{\partial B_{z}^{\text {coil }}}{\partial r} /\left[\frac{\partial B_{z}^{\text {iron }}}{\partial r}+\frac{\partial B_{z}^{\text {coil }}}{\partial r}\right] \tag{2}
\end{equation*}
$$

Eq. (2) clearly confirms the dominance of the coil field for $r<78 \mathrm{~cm}$. Fig. 1c shows the iso- Br contours for a case where the coil has been shifted by $\delta=2 \mathrm{~mm}$. For $r<78 \mathrm{~cm}$, the median plane also shifts 2 mm . At $r=80.3 \mathrm{~cm}$, the denominator of Eq. (2) becomes zero, which explains the strong shift of the median plane just before this radius.


Fig. 1: a) azimuthally averaged fields of the coil, the iron and their sum, b) the radial components 1 mm from the median plane, c) contours of equal total Br for a coil shift of 2 mm calculated using Eq. (2), d) median plane position due to a coil shift of 1 mm calculated from Eq. (4) together with $v_{z \text {. }}$ The dashed line at $r=78 \mathrm{~cm}$ is at minimum $v_{z}$.

In Ref. [2], a relation is given between field properties such as the vertical oscillation frequency $v_{z}$ and the median plane position. This can be derived by using the relation between $B$ and $v_{z}$ :

$$
\begin{equation*}
v_{z}^{2}=-\frac{r}{B_{z}^{\text {total }}} \cdot \frac{\partial B_{z}^{\text {total }}}{\partial r}+F \tag{3}
\end{equation*}
$$

in which we have not neglected the flutter $F$, since this factor is large near the extraction radius. Inserting Eq. (3) into Eq. (2) then gives the average median plane position $Z$ as function of radius, due to a shift $\delta$ of the coil:

$$
\begin{equation*}
Z(r)=\frac{\partial B_{z}^{c o i l}}{\partial r} \cdot \frac{r}{B_{z}} \cdot \frac{\delta}{-v_{z}^{2}+F} \tag{4}
\end{equation*}
$$

It should be noted that when the flutter is neglected, Eq. (4) is the same as the one given in Ref. [2]. However, neglecting the flutter in the calculations would yield a singularity in the median plane position at $r=80.3 \mathrm{~cm}$ where $\partial B_{z} / \partial r=0$. When we replace the last denominator in Eq. (4) by values of $-v_{z}^{2}$ derived from the transfer matrix, which is calculated by particle tracking [3], we obtain the curve shown in Fig. 1d for $\delta=1 \mathrm{~mm}$. In this figure, the values used for $v_{z}$ are also plotted.

## COIL-MISALIGNMENT EFFECTS ON THE BEAM

The beam behavior due to a median plane shift has been investigated with a 3D-simulation using the raytracing program TRACK [1]. The beam energy gain per turn and the beam radius increase per turn are only weakly influenced by a vertical coil shift of $\delta=0.1 \mathrm{~mm}$, as can be expected. All three values are shown in Fig. 2. However, the vertical beam position shows a strong effect. It is remarkable that the vertical position of the beam (started at $z=0$ ) shows the same behavior as the analytical calculation of the median plane shift shown in Fig. 1d.

An important finding (in agreement with [2]) is that the maximum shift of the beam is roughly 25 times larger than the vertical coil shift itself. Eq. (4) and Fig. 1d suggest that this shift is very sensitive to $v_{z}$. Close to the extraction radius ( $r=81.5 \mathrm{~cm}$ ), the radial field changes its sign and thereafter the strong vertical focusing almost completely corrects the beam offset. It is interesting to note that the beam does not "follow" the median plane (Eq. 2) itself, but that the shift causes a slow movement of the beam position with an amplitude that also depends on $v_{z}$ and similar to the results of the analytical formula shown in Fig. 1d.

We expect that the concept of using an analytically derived value of an azimuthally averaged $v_{z}$ may not be fully valid near extraction, where the field changes are dramatic and the orbits deviate strongly from circles. Nevertheless, by introducing appropriate flutter terms, we strive to improve the analytical formula.


Fig. 2: Comparing energy gain, radius increase per turn and the vertical deviation of the beam, for symmetrically positioned and vertically displaced coils.

In order to investigate whether the beam size also changes, we traced a beam of finite size. As can be expected, almost no effect due to the coil shift is observed in the horizontal plane. The effect in the vertical direction is shown in Fig. 3. Although the beam position varies with radius, the vertical beam size is not changing much, even where the beam makes a large excursion.


Fig. 3: Vertical particle positions for different beam starting conditions. The coil was shifted 0.1 mm

## CONCLUSIONS

The particle tracking confirmed the importance of proper coil alignment and confirmed results from other cyclotrons. Work is in progress to make a link with analytical models, aiming at a better understanding of the phenomena.

## REFERENCES

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