Orbit Correction Within Constrained Solution Spaces

- 6 horizontal correctors
- 26 vertical correctors
Constraints for Orbit Correction

i. Hardware limitation of physical corrector strengths
   - Global orbit correction on a misaligned (low emittance) magnet lattice
   - Local orbit bumps of increased amplitude (aperture scans, BBC, etc)
   - Enhanced by:
     • Little maximum corrector strengths
     • Little Phase advances (low tune)
     • Calibrational errors of BPM offsets

ii. Solution space of local impact (bumps)

iii. Exploitation of nullspace

DELTA Corrector Design

Corrector Coils
Magnetic Field
Electron Beam
Resulting Deflection
Corrector Strength

- **horizontal correctors:**
  
  - long yoke (40cm):
    
    2x150 windings
    
    max: $0-1.8$ mrad @ 1.5 GeV
  
  - short yoke (20cm):
    
    2x240 windings
    
    max: $0-1.5$ mrad @ 1.5 GeV

- **vertical correctors:**
  
  - short yokes only:
    
    4x50 windings
    
    max: $1.1 - 0.5$ mrad @ 1.5 GeV

### Constraints for Orbit Correction

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**Vertical BPM Phase Advances at DELTA**

**Phase advance between neighbouring BPMs \[\pi\]**

**Total Phase Advance \[2\pi\]**

**Highly sensitive to calibration!**

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**Significance of Calibration Errors**

**Correction of simulated offsets for:**
- \(\text{bpm16} - 100\mu\text{m}\)
- \(\text{bpm17} + 100\mu\text{m}\)

- **Large corrector strengths afforded** (>0.5 out of 0.5 to 1.1 mrad max)
- **Error propagation** (1.2 mm !!)

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SVD-Based Orbit Correction

\[
\begin{pmatrix}
w_1 & 0 \\
\vdots & \ddots \\
0 & w_m
\end{pmatrix}
\begin{pmatrix}
\tilde{R}_1 \\
\tilde{R}_2 \\
\vdots \\
\tilde{R}_n
\end{pmatrix} = WR = U[\text{diag}(\sigma_i)]V^T
\]

\[U^T U = V^T V = VV^T = 1\]

Inverse within the Range of WR (Pseudoinverse):

\[(WR)^\# = V[\text{diag}(1/\sigma_i)]U^T\]

\[\tilde{\theta}_{OC} = -R^\# \Delta \tilde{\kappa}\]

\[\left\|W(R\tilde{\theta}_{OC} + \Delta \tilde{\kappa})\right\|_2 \rightarrow \text{min.}\]
DOF* for Correction

Free Parameter for SVD-Based matrix inversion:

\[(WR)^\# = V^T [\text{diag}(1/\sigma_i)] U\]

\[1/\sigma_i \leftarrow 0 \quad \forall \quad \sigma_i \leq \sigma_{\text{cutoff}}\]

Relative corrector strength

-100% +100%

• rather fictitious task here
• typical for DELTA about 2 limited correctors vertically (usually no problems horizontally)
Weighted Orbit Space

→ Find the closest spot to a given point on a set of hyperplanes in n dimensions

\[ \left\| U^T W (R \tilde{\theta}_{oc} + \Delta \tilde{K}) \right\|_2 \rightarrow \min. \]

→ Restrict solution space to the "Range"

\[ \text{span} \{ \bar{u}_i | \sigma_i > \sigma_{cutoff} \} \]

→ n-dimensional solving strategies...

→ KKT criterion to identify unique solution

Marc Grewe, INBS 2004, Grindelwald, CH
Test of Code Integrity

Residual orbit deviation

Quality of correction

DOF for correction

Test of Code Integrity

Residual orbit deviation

Quality of correction

DOF for correction

No correction

worse

# of limiting correctors

better

Marc Grewe, IWBS 2004, Grindelwald, CH

Constraints for Local Orbit Bumps

• Locality = minimise orbit impact outside bump (ROI)
  - Choose correctors $k$ surrounding bump monitors $\mathcal{H} = \{k\}$
  
  \[ \text{Solve} \quad \left\| U_B^T W_B \left( R_B \tilde{\theta}_B + \tilde{\kappa}_{B,\text{ref}} \right) \right\|_2 \to \min. \]

  ... under restriction of the solution space to

  \[ \text{span}\{\bar{u}_{B,i} | \sigma_{B,i} > \sigma_{\text{cutoff}}\} \cap \text{span}\{ U_B^T W_B R_B \bar{V}_{B,\text{ref}}^* | \bar{V}_{B,i}^* \in V_B^*, i > 2\} \]

  \[- |H| - 2 \text{ column vectors } V_{B,\text{cutoff}} \text{ of } V_B^* \text{ corresponding to remaining SVs constitute ON-Basis for local orbit bumps} \]

  (minimum impact outside ROI)

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Local Orbit Bumps (Example)

- Dynamically choose as many correctors as needed
- Produce asked orbit offset, without concern, of corr. lims.
- Provided as agent service to external clients

Exploitation of Nullspace (i)

- Use little orbit impact of ON-Base

\[ \text{span}\left\{ \tilde{V}_i | \tilde{V}_i \in V, \sigma_i \leq \sigma_{\text{cutoff}} \right\} \]

to ease large corrector strengths in current setting \( \bar{\theta}_0 \):

\[ \text{find} \ \bar{\theta}_c \ \text{such as to minimise} \]

\[ \left\| W_C (\bar{\theta}_c + \bar{\theta}_0) \right\|_2 \rightarrow \min. \]

with diagonal corrector weight matrix

\[ W_C := \begin{pmatrix} w_1^c & 0 \\ \vdots & \ddots \\ 0 & w_n^c \end{pmatrix} \]

\( \leftarrow \) choose large weights \( w_j^c \)

to put an emphasis on correctors \( j \) to be eased
Exploitation of Nullspace (ii)

- ... again, use SVD to create a weighted ON-Base $\mathbf{U}_c$ of correctors:

$$\mathbf{W} = \mathbf{U} \left[ \text{diag}(\sigma_c) \right] \mathbf{V}_c^T$$

→ Solve

$$\| \mathbf{U}_c^T \mathbf{W}_c (\tilde{\theta}_c + \tilde{\theta}_0) \|_2 \to \min.$$  

... under restriction of the solution space to

$$\text{span}\{ \mathbf{U}_c^T \mathbf{W}_c \mathbf{V}_i \ | \ \sigma_i \leq \sigma_{\text{cutoff}} \}$$

Bottom Line

SVD based orbit correction restricted to the common set of feasible corrector strengths

- Flexibility
  - (no matter what the offsets and limits are)

- Effectivity (!?)
  - (still an idea, usefulness to be proven)

- Reliability!!
  - (while retaining the best solution possible)

Restriction to local orbit bumps

Restriction to Nullspace

operational since 4/2004
Thanks for your attention