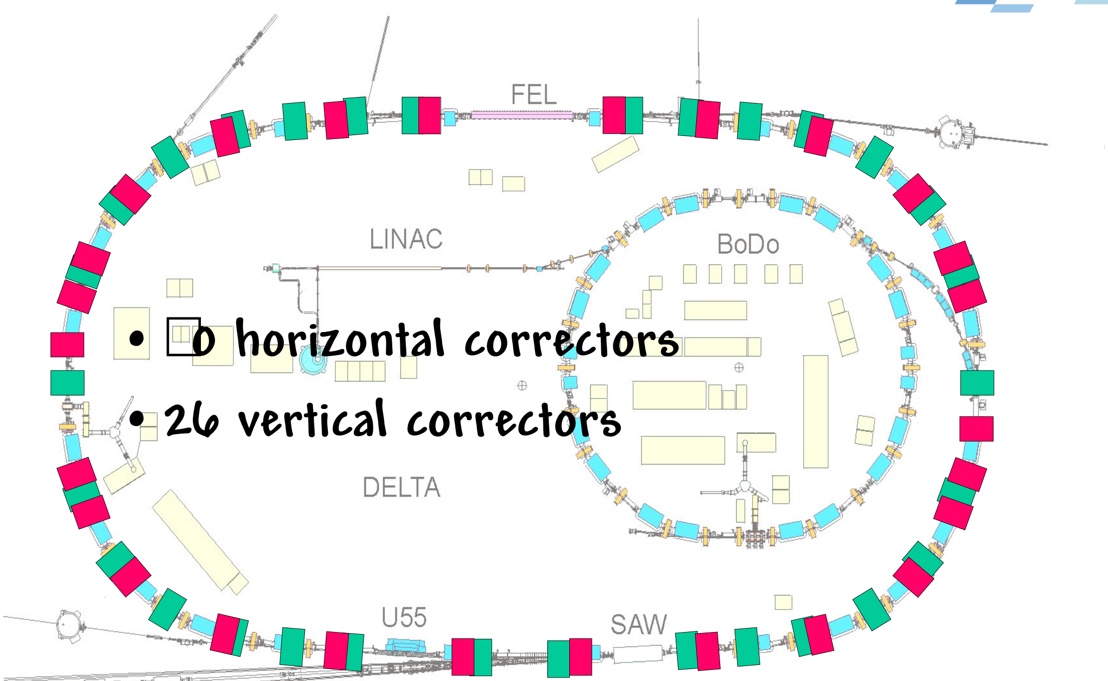
Orbit Correction Within Constrained Solution Spaces



- □0 horizontal correctors
- 26 vertical correctors

Constraints for Orbit Correction

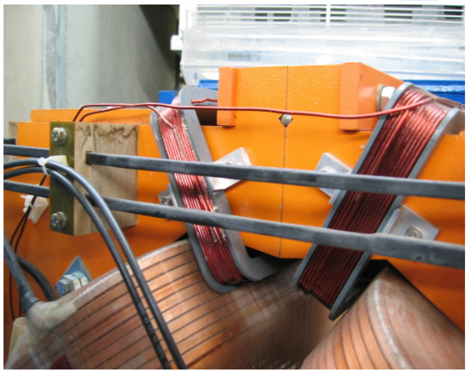
i. Hardware limitation of physical corrector strengths

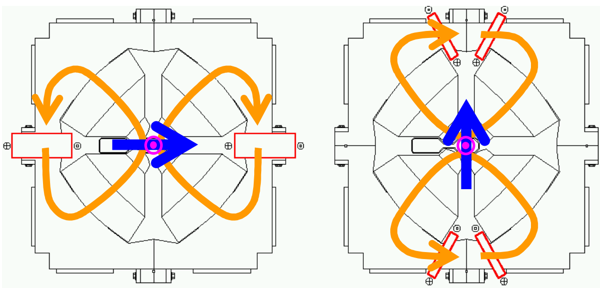
- Global orbit correction on a misaligend (low emittance) magnet lattice
- Local orbit bumps of increased amplitude (aperture scans, BBC, etc)
- Enhanced by:
  - Little maximum corrector strengths
  - Little Phase advances (low tune)
  - Calibrational errors of BPM offsets

ii. Solution space of local impact (bumps)

iii. Exploitation of nullspace

DELTA Corrector Design

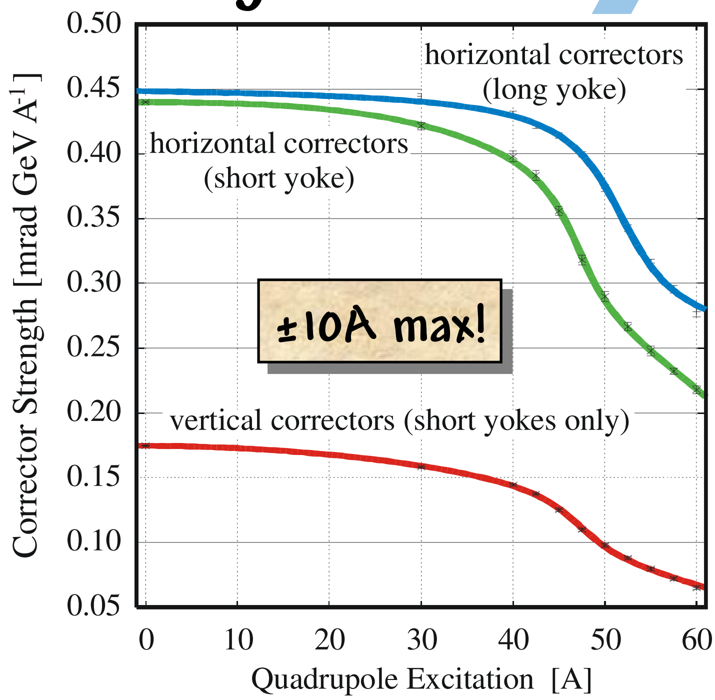




Corrector Coils
Magnetic Field
Electron Beam
Resulting Deflection

Corrector Strength

- horizontal correctors:
  - long yoke (40cm): 2x150 windings
    max: □0 - 1.8 mrad @ 1.5 GeV
  - short yoke (20cm): 2x240 windings
    max: □0 - 1.5 mrad @ 1.5 GeV
- vertical correctors:
  - short yokes only: 4x50 windings
    max: 1.1 - 0.5 mrad @ 1.5 GeV



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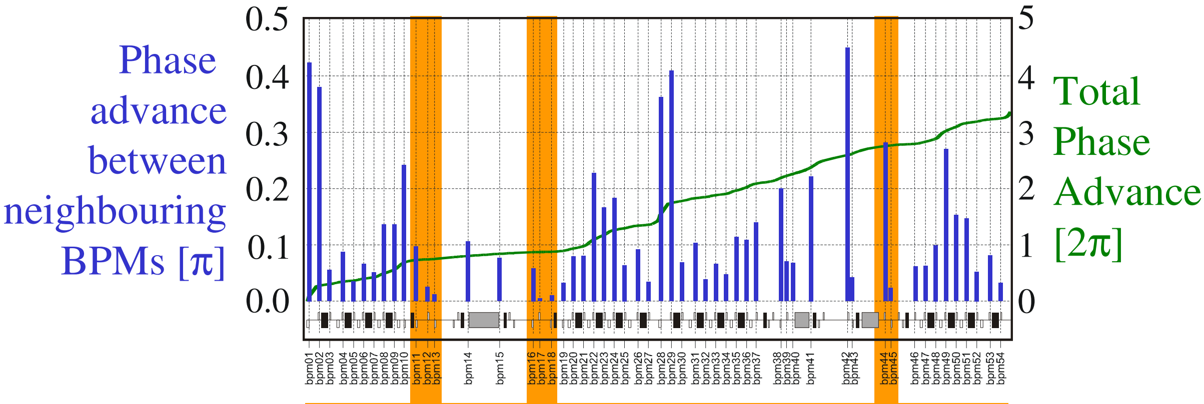
Vertical BPM Phase Advances at DELTA



Highly sensitive to calibration!

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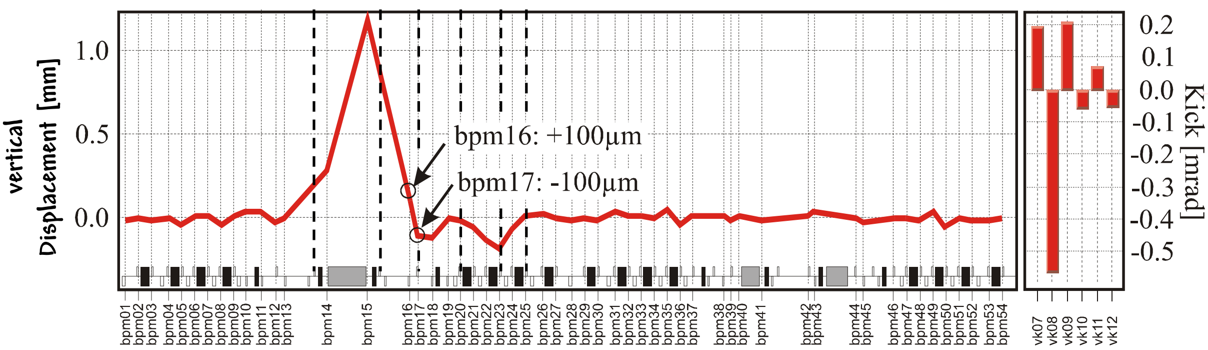
Significance of Calibration Errors

Correction of simulated offsets for: bpm16 -100µm, bpm17 +100µm



- Large corrector strengths afforded (>0.5 out of 0.5 to 1.1 mrad max)
- Error propagation (1.2mm !!)

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SVD-Based Orbit Correction

$$\begin{pmatrix} w_1 & & 0 \\ & \ddots & \\ 0 & & w_m \end{pmatrix} \begin{pmatrix} \vec{R}_1 \vec{R}_2 \vec{R}_3 \cdots \vec{R}_n \end{pmatrix} =: \mathbf{WR} =: \mathbf{U}[\mathrm{diag}(\sigma_i)]\mathbf{V}^{\mathrm{T}}$$

$$\mathbf{U}^{\mathrm{T}}\mathbf{U} = \mathbf{V}^{\mathrm{T}}\mathbf{V} = \mathbf{V}\mathbf{V}^{\mathrm{T}} = \mathbf{1}$$

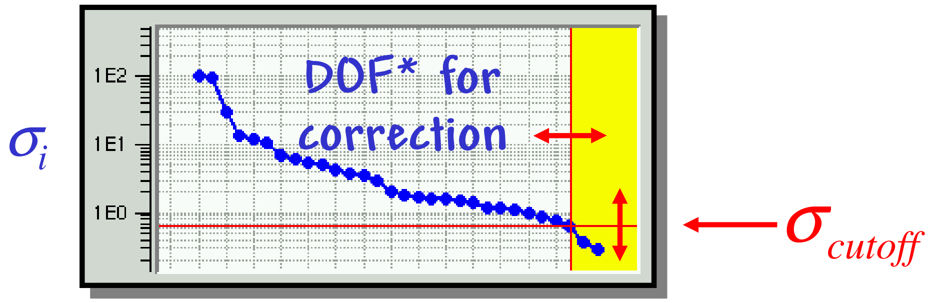
Inverse within the Range of WR (Pseudoinverse):	$(\mathbf{WR})^{\#} = \mathbf{V}[\mathrm{diag}(1/\sigma_i)]\mathbf{U}^{\mathrm{T}}$ $\vec{\theta}_{OC} = -\mathbf{R}^{\#}\Delta\vec{\kappa}$ $\left\|\mathbf{W}(\mathbf{R}\vec{\theta}_{OC} + \Delta\vec{\kappa})\right\|_2 \to \min.$

DOF* for Correction

Free Parameter for SVD-Based matrix inversion:

$$(\mathbf{WR})^{\#} = \mathbf{V}^{T}\left[\mathrm{diag}(1/\sigma_i)\right]\mathbf{U}$$

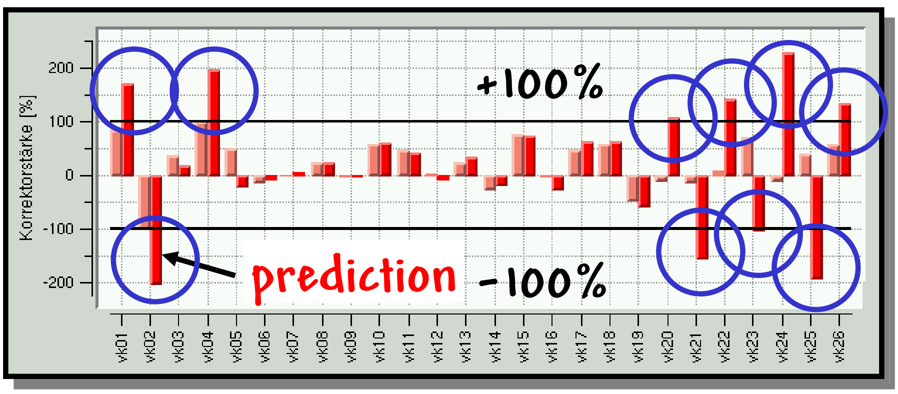
$$1/\sigma_i \leftarrow 0 \quad \forall \quad \sigma_i \leq \sigma_{cutoff}$$



*DOF= Degrees Of Freedom

Solutions for Constraining Correctors ?

Relative corrector strength



- rather ficticious task here
- typical for DELTA about 2 limited correctors vertically (usually no problems horizontally)

Weighted Orbit Space

→ Find the closest spot to a given point on a set of hyperplanes in **n dimensions**

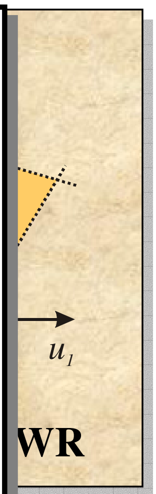
$$\left\| \mathbf{U}^{\mathrm{T}} \mathbf{W} \left(\mathbf{R} \vec{\theta}_{OC} + \Delta \vec{\kappa} \right) \right\|_2 \rightarrow \min .$$

→ Restrict solution space to the „Range"

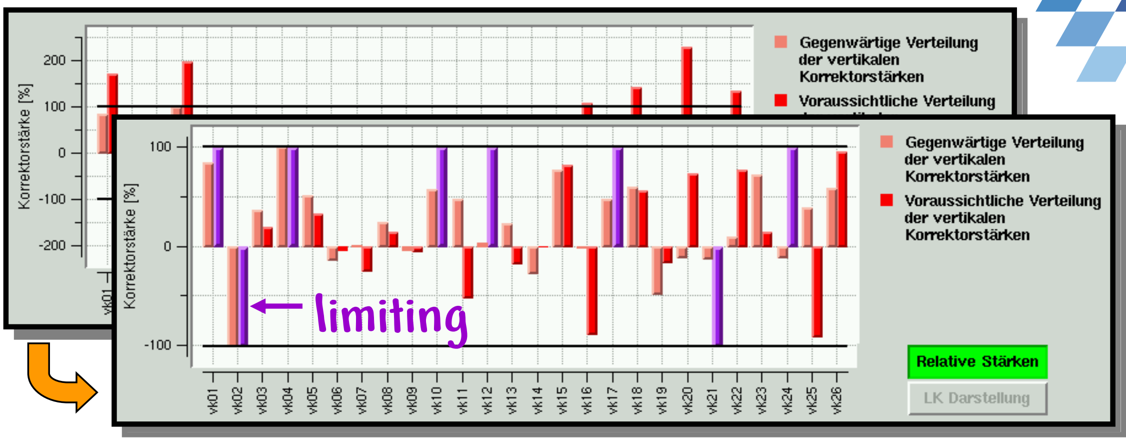
$$\operatorname{span}\left\{ \vec{u}_i \mid \sigma_i > \sigma_{cutoff} \right\}$$

→ n-dimensional solving strategies...

→ KKT criterion to identify **unique** solution



C... Space | ... Space
intermingled nullspace | orthogonal nullspace



! Numbers scale with weights !

		→	
$<z\text{-}z_{ref}>$	+0.22 ± 0.64 mm	→	+0.02 ± 0.10 mm
$<z\text{-}z_{ref}>_v$	+0.00 ± -.--	→	+0.00 ± -.--
χ^2_z	3.07e+06	→	3.08e+04 (1.0 %)

		→	
$<z\text{-}z_{ref}>$	+0.22 ± 0.64 mm	→	+0.02 ± 0.13 mm
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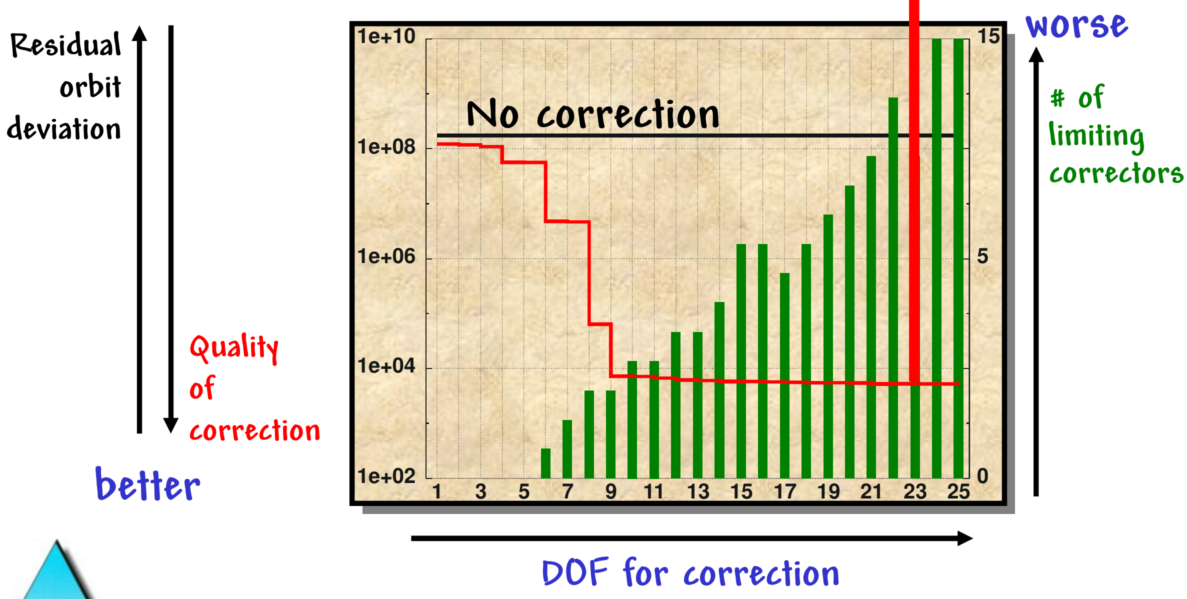
χ^2_z	3.07e+06	→	3.11e+04 (1.0 %)

‚suboptimal' strategy

Test of Code Integrity



Constraints for Local Orbit Bumps

- Locality = minimise orbit impact ouside bump (ROI)
  - Choose correctors k surrounding bump monitors $\rightarrow \mathcal{H} = \{k\}$

→Solve

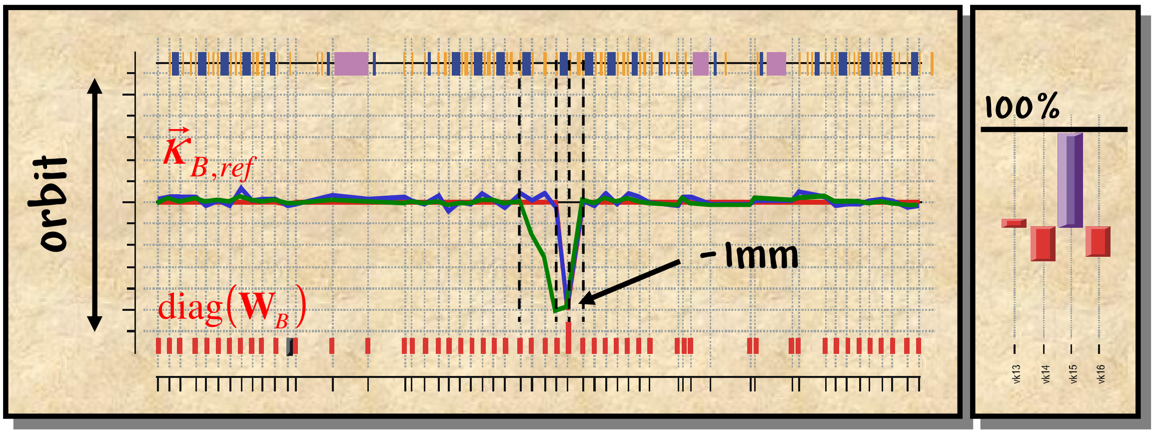
$$\left\| \mathbf{U}_B^T \mathbf{W}_B \left(\mathbf{R}_B \vec{\theta}_B + \vec{\kappa}_{B,ref} \right) \right\|_2 \rightarrow \min .$$

... under restriction of the solution space to

$$\mathrm{span}\left\{ \vec{u}_{B,i} \mid \sigma_{B,i} > \sigma_{cutoff} \right\} \cap \mathrm{span}\left\{ \mathbf{U}_B^T \mathbf{W}_B \mathbf{R}_B \vec{V}^*_{B,i} \mid \vec{V}^*_{B,i} \in \mathbf{V}^*_B, i > 2 \right\}$$

  - $|\mathcal{H}|$-2 column vectors $V^*_{B,i>2}$ of $\mathbf{V}^*_B$ corresponding to remaining SVs constitute ON-Basis for local orbit bumps (minimum impact outside ROI)

Local Orbit Bumps (Example)



- Dynamically choose as many correctors as needed
- Produce asked orbit offset ‚without concern' of corr. lims.
- Provided as agent service to external clients

Exploitation of Nullspace (i)

- Use little orbit impact of ON-Base

$$\operatorname{span}\left\{ \vec{V}_i \mid \vec{V}_i \in \mathbf{V}, \sigma_i \leq \sigma_{cutoff} \right\}$$

to ease large corrector strengths in current setting $\vec{\theta}_0$:

$\rightarrow$ find $\vec{\theta}_C$ such as to minimise

$$\left\| \mathbf{W}_C \left(\vec{\theta}_C + \vec{\theta}_0 \right) \right\|_2 \rightarrow \min .$$

with diagonal corrector weight matrix

$$\mathbf{W}_C := \begin{pmatrix} w_1^C & & 0 \\ & \ddots & \\ 0 & & w_n^C \end{pmatrix}$$

$\leftarrow$ choose large weights w_j^C to put an emphasis on correctors j to be eased

Exploitation of Nullspace (ii)

- … again, use SVD to create a weighted ON-Base $\mathbf{U}_C$ of correctors:

$$\mathbf{W}_C = \mathbf{U}_C \left[\mathrm{diag}\left(\sigma^C\right)\right] \mathbf{V}^{\mathrm{T}}$$

→ Solve

$$\left\| \mathbf{U}_C^{\mathrm{T}} \mathbf{W}_C \left(\vec{\theta}_C + \vec{\theta}_0\right) \right\|_2 \rightarrow \min .$$

… under <u>restriction</u> of the solution space to

$$\mathrm{span}\left\{ \mathbf{U}_C^{\mathrm{T}} \mathbf{W}_C \vec{V}_i \mid \sigma_i \leq \sigma_{cutoff} \right\}$$

Bottom Line

SVD based orbit correction restricted to the common set of feasable corrector strengths

- + Restriction to local orbit bumps → **Flexibility** („no matter" what the offsets and limits are)
- + Restriction to Nullspace → **Effectivity (!?)** (still an idea, usefulness to be proven)
- operational since 4/2004 → **Reliability!!** (while retaining the best solution possible)

Thanks for your attention

Monitor Positioning